From the Neutrino to the Edge of the Universe

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Abstract

Two recent findings necessitate a closer look at the existing standard models of Particle Physics and Cosmology. These are the discovery of Neutrino oscillation, and hence a non zero mass on the one hand and, on the other, observations of distant supernovae which indicate that contrary to popular belief, the universe would continue to expand for ever, possibly accelerating in the process. In this paper it is pointed out that relatively recent studies which indicate a stochastic, quantum vacuum underpinning and a fractal structure for space time, reconcile both of the recent observations, harmoniously.

1 Introduction

In the recent years, there have been two significant findings which necessitate a closer look at the existing standard models of Particle Physics and Cosmology. The first is the Superkamiokande experiment[1] which demonstrates a neutrino oscillation and therefore a non zero mass, whereas, strictly going by the standard model, the neutrino should have zero mass. The other finding based on distant supernovae observations[2, 3, 4] is that the universe will continue to expand without deceleration and infact possibly accelerating in the process.

We will now demonstrate how a recent model of fractal, quantized space time

[0] Partly based on an invited talk at the National Workshop on Neutrino Physics, University of Hyderabad, 1998.

arising from the underpinning of a quantum vaccuum or Zero Point Field, reconciles both the above facts, in addition to being in agreement with other experimental and observational data.

2 Neutrino Mass

According to a recent model, elementary particles, typically leptons, can be treated as, what may be called Quantum Mechanical Black Holes (QMBH)[5, 6, 7, 8, 9], which share certain features of Black Holes and also certain Quantum Mechanical characteristics. Essentially they are bounded by the Compton wavelength within which non local or negative energy phenomena occur, these manifesting themselves as the Zitterbewegung of the electron. These Quantum Mechanical Black Holes are created out of the background Zero Point Field and this leads to a consistent cosmology, wherein using N, the number of particles in the universe as the only large scale parameter, one could deduce from the theory, Hubble's law, the Hubble's constant, the radius, mass, and age of the universe and features like the hitherto inexplicable relation between the pion mass and the Hubble constant[5]. The model also predicts an ever expanding universe, as recent observations do confirm.
Within this framework, it was pointed out that the neutrino would be a massless and charge less version of the electron and it was deduced that it would be lefthanded, because one would everywhere encounter the psuedo spinorial ("negative energy") components of the Dirac spinor, by virtue of the fact that its Compton wavelength is infinite (in practise very large). Based on these considerations we will now argue that the neutrino would exhibit an anomalous Bosonic behaviour which could provide a clue to the neutrino mass.
As detailed in [6] the Fermionic behaviour is due to the non local or Zitterbewegung effects within the Compton wavelength effectively showing up as the well known negative energy components of the Dirac spinor which dominate within while positive energy components predominate outside leading to a doubly connected space or equivalently the spinorial or Fermionic behaviour. In the absence of the Compton wavelength boundary, that is when we encounter only positive energy or only negative energy solutions, the particle would not exhibit the double valued spinorial or Fermionic behaviour: It would have an anomalous anyonic behaviour.

Indeed, the three dimensionality of space arises from the spinorial behaviour outside the Compton wavelength[10]. At the Compton wavelength, this disappears and we should encounter lower dimensions. As is well known[11] the low dimensional Dirac equation has like the neutrino, only two components corresponding to only one sign of the energy, displays handedness and has no invariant mass. The neutrino shows up as a fractal entity.
Ofcourse the above model strictly speaking is for the case of an isolated non interacting particle. As neutrinos interact through the weak or gravitational forces, both of which are weak, the conclusion would still be approximately valid particularly for neutrinos which are not in bound states.
We will now justify the above conclusion from other standpoints: Let us first examine why Fermi-Dirac statistics is required in the Quantum Field Theoretic treatment of a Fermion satisfying the Dirac equation. The Dirac spinor has four components and there are four independent solutions corresponding to positive and negative energies and spin up and down. It is well known that [12] in general the wave function expansion of the Fermion should include solutions of both signs of energy:

$$\psi(\vec{x}, t) = N \int d^3p \sum_{\pm s} [b(p,s)u(p,s)exp(-\imath p^\mu x_\mu/\hbar) \\ + d^*(p,s)v(p,s)exp(+\imath p^\mu x_\mu/\hbar) \quad (1)$$

where N is a normalization constant for ensuring unit probability.
In Quantum Field Theory, the coefficients become creation and annihilation operators while bb^+ and dd^+ become the particle number operators with eigen values 1 or 0 only. The Hamiltonian is now given by[13]:

$$H = \sum_{\pm s} \int d^3p E_p [b^+(p,s)b(p,s) - d(p,s)d^+(p,s)] \quad (2)$$

As can be seen from (2), the Hamiltonian is not positive definite and it is this circumstance which necessitates the Fermi-Dirac statistics. In the absence of Fermi-Dirac statistics, the negative energy states are not saturated in the Hole Theory sense so that the ground state would have arbitrarily large negative energy, which is unacceptable. However Fermi-Dirac statistics and the anti commutators implied by it prevent this from happening.
From the above, it follows that as only one sign of energy is encountered for the v, we need not take recourse to Fermi-Dirac statistics.

We will now show from an alternative view point also that for the neutrino, the positive and negative solutions are delinked so that we do not need the negative solutions in (1) or (2) and there is no need to invoke Fermi-Dirac statistics.
The neutrino is described by the two component Weyl equation[14]:

$$\imath\hbar\frac{\partial\psi}{\partial t} = \imath\hbar c\vec{\sigma}\cdot\vec{\Delta}\psi(x) \tag{3}$$

It is well known that this is equivalent to a massless Dirac particle satisfying the following condition:

$$\Gamma_5\psi = -\psi$$

We now observe that in the case of a massive Dirac particle, if we work only with positive solutions for example, the current or expectation value of the velocity operator $c\vec{\alpha}$ is given by (ref.[12]),

$$J^+ =< c\alpha >=< \frac{c^2\vec{p}}{E} > + =< v_{gp} > + \tag{4}$$

in an obvious notation.
(4) leads to a contradiction: On the one hand the eigen values of $c\vec{\alpha}$ are $\pm c$. On the other hand we require, $< v_{gp} >< 1$.
To put it simply, working only with positive solutions, the Dirac particle should have the velocity c and so zero mass. This contradiction is solved by including the negative solutions also in the description of the particle. This infact is the starting point for (1) above.
In the case of mass less neutrinos however, there is no contradiction because they do indeed move with the velocity of light. So we need not consider the negative energy solutions and need work only with the positive solutions. There is another way to see this. Firstly, as in the case of massive Dirac particles, let us consider the packet (1) with both positive and negative solutions for the neutrino. Taking the z axis along the $\vec{p}$ direction for simplicity, the acceptable positive and negative Dirac spinors subject to the above stated condition are

$$u = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} v = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

The expression for the current is now given by,

$$J^z = \int d^3p \left\{ \sum_{\pm s} [|b(p,s)|^2 + |d(p,s)|^2] \frac{p^z c^2}{E} \right.$$
$$+\imath \sum_{\pm s \pm s'} b^*(-p,s')d^*(p,s) - \bar{u}(-p,s')\sigma^{30}v(p,s)$$
$$\left. -\imath \sum_{\pm s \pm s'} b(-p,s')d(p,s) - \bar{v}(p,s')\sigma^{30}u(-p,s) \right\} \quad (5)$$

Using the expressions for u and v it can easily be seen that in (5) the cross (or Zitterbewegung) term disappears.
Thus the positive and negative solutions stand delinked in contrast to the case of massive particles, and we need work only with positive solutions (or only with negative solutions) in (1).
Finally this can also be seen in yet another way. As is known (ref.[14]), we can apply a Foldy-Wothuysen transformation to the mass less Dirac equation to eliminate the "odd" operators which mix the components of the spinors representing the positive and negative solutions.
The result is the Hamiltonian,

$$H' = \Gamma^\circ pc \quad (6)$$

Infact in (6) the positive and negative solutions stand delinked. In the case of massive particles however, we would have obtained instead,

$$H' = \Gamma^\circ \sqrt{(p^2c^2 + m_0c^4)} \quad (7)$$

and as is well known, it is the square root operator on the right which gives rise to the "odd" operators, the negative solutions and the Dirac spinors. Infact this is the problem of linearizing the relativistic Hamiltonian and is the starting point for the Dirac equation.
Thus in the case of mass less Dirac particles, we need work only with solutions of one sign in (1) and (2). The equation (2) now becomes,

$$H = \sum_{\pm s} \int d^3pE_p[b^+(p,s)b(p,s)] \quad (8)$$

As can be seen from (8) there is no need to invoke Fermi-Dirac statistics now. The occupation number bb^+ can now be arbitrary because the question of

a ground state with arbitrarily large energy of opposite sign does not arise. That is, the neutrinos obey anomalous statistics.
In a rough way, this could have been anticipated. This is because the Hamiltonian for a mass less particle, be it a Boson or a Fermion, is given by

$$H = pc$$

Substitution of the usual operators for H and p yields an equation in which the wave function ψ is a scalar corresponding to a Bosonic particle.
According to the spin-statistics connection, microscopic causality is incompatible with quantization of Bosonic fields using anti-commutators andr Fermi fields using commutators[13]. But it can be shown that this does not apply when the mass of the Fermion vanishes.
In the case of Fermionic fields, the contradiction with microscopic causality arises because the symmetric propogator, the Lorentz invariant function,

$$\Delta_1(x-x') \equiv \int \frac{d^3k}{(2\pi)^3 3\omega_k}[e^{-\imath k.(x-x')} + e^{\imath k.(x-x')}]$$

does not vanish for space like intervals $(x-x')^2 < 0$, where the vacuum expectation value of the commutator is given by the spectral representation,

$$S_1(x-x') \equiv \imath < 0|[\psi_\alpha(x), \psi_\beta(x')]|0 >= -\int dM^2[\imath\rho_1(M^2)\Delta_x + \rho_2(M^2)]_{\alpha\beta}\Delta_1(x-x')$$

Outside the light cone, $r > |t|$, where $r \equiv |\vec{x} - \vec{x}'|$ and $t \equiv |x_0 - x_0'|, \Delta_1$ is given by,

$$\Delta_1(x'-x) = -\frac{1}{2\pi^2 r}\frac{\partial}{\partial r}K_0(m\sqrt{r^2-t^2}),$$

where the modified Bessel function of the second kind, K_0 is given by,

$$K_0(mx) = \int_0^\infty \frac{cos(xy)}{\sqrt{m^2+y^2}}dy = \frac{1}{2}\int_{-\infty}^\infty \frac{cos(xy)}{\sqrt{m^2+y^2}}dy$$

(cf.[15]). In our case, $x \equiv \sqrt{r^2-t^2}$, and we have,

$$\Delta_1(x-x') = const\frac{1}{x}\int_{-\infty}^\infty \frac{ysinxy}{\sqrt{m^2+y^2}}dy$$

As we are considering massless neutrinos, going to the limit as $m \to 0$, we get, $|Lt_{m\to 0}\Delta_1(x-x')| = |(const.).Lt_{m\to 0}\frac{1}{x}\int_{-\infty}^\infty sinxydy| < \frac{0(1)}{x}$. That is, as

the Compton wavelength for the neutrino is infinite (or very large), so is $|x|$ and we have $|\Delta_1| << 1$. So the invariant Δ_1 function nearly vanishes everywhere except on the light cone $x = 0$, which is exactly what is required. So, the spin-statistics theorem or microscopic causality is not violated for the mass less neutrinos when commutators are used.

The fact that the ideally, massless, spin half neutrino obeys anomalous statistics could have interesting implications. For, given an equilibrium collection of neutrinos, we should have if we use the Bose-Einstein statistics[16].

$$PV = \frac{1}{3}U, \tag{9}$$

instead of the usual

$$PV = \frac{2}{3}U, \tag{10}$$

where P, V and U denote the pressure, volume and energy of the collection. We also have, $PV \alpha NkT, N$ and T denoting the number of particles and temperature respectively.

On the other hand for a fixed temperature and number of neutrinos, comparison of (9) and (10) shows that the effective energy U' of the neutrinos would be twice the expected energy U. That is in effect the neutrino acquires a rest mass m. It can easily be shown from the above that,

$$\frac{mc^2}{k} \leq\approx \sqrt{3}T \tag{11}$$

That is for cold background neutrinos m is about a thousandth of an ev at the present background temperature of about $2^\circ K$:

$$10^{-9}m_e \leq m \leq 10^{-8}m_e \tag{12}$$

This can be confirmed, alternatively, as follows. As pointed out by Hayakawa, the balance of the gravitational force and the Fermi energy of these cold background neutrinos, gives[17],

$$\frac{GNm^2}{R} = \frac{N^{2/3}\hbar^2}{mR^2}, \tag{13}$$

where N is the number of neutrinos.

Further as in the Kerr-Newman Black Hole formulation equating (13) with the energy of the neutrino, mc^2 we immediately deduce

$$m \approx 10^{-8}m_e$$

which agrees with (11) and (12). It also follows that $N \sim 10^{90}$, which is correct. Moreover equating this energy of the quantum mechanical black hole to kT, we get (cf.also (11))

$$T \sim 1^\circ K,$$

which is the correct cosmic background temperature.
Alternatively, using (11) and (12) we get from (13), a background radiation of a few millimeters wavelength, as required.
So we obtain not only the correct mass and the number of the neutrinos, but also the correct cosmic background temperature, at one stroke.
Indeed the above mass of the neutrino was predicted earlier[18].

3 Cosmology

The above model of quantized space time ties up with the model of fluctuational cosmology discussed in several papers[8].
We observe that the ZPF leads to divergences in QFT[19] if no large frequency cut off is arbitrarily prescribed, e.g. the Compton wavelength. We argue that it is these fluctuations within the Compton wavelength and in time intervals $\sim \hbar/mc^2$, which create the particles. Thus choosing the pion as a typical particle, we get[19, 5],

$$(\text{Energy} \quad \text{density} \quad \text{of} \quad \text{ZPF}) X l^3 = mc^2 \tag{14}$$

Using the fact there are $N \sim 10^{80}$ such particles in the Universe, we get,

$$Nm = M \tag{15}$$

where M is the mass of the universe.
We equate the gravitational potential energy of the pion in a three dimensional isotropic sphere of pions of radius R, the radius of the universe, with the rest energy of the pion, to get,

$$R = \frac{GM}{c^2} \tag{16}$$

where M can be obtained from (15).
We now use the fact that the fluctuation in the particle number is of the order

$\sqrt{N}$[17, 16, 5], while a typical time interval for the fluctuations is $\sim \hbar/mc^2$ as seen above. This leads to the relation[5]

$$T = \frac{\hbar}{mc^2}\sqrt{N} \tag{17}$$

where T is the age of the universe, and

$$\frac{dR}{dt} \approx HR \tag{18}$$

Strictly speaking the above equations are order of magnitude relations. So from (18), a further differenciation leads to the conclusion that a cosmological constant cannot be ruled out such that

$$\Lambda \approx \leq 0(H^2) \tag{19}$$

(19) explains the smallness of the cosmological constant or the so called cosmological problem[20].
To proceed it can be shown that the above equations lead to[21]

$$G = \frac{\beta}{T} \equiv G_0(1 - \frac{t}{t_0}) \tag{20}$$

where t_0 is the age of the universe and T is the time that has elapsed in the present epoch. It can be shown that (20) can explain the precession of the perihelion of Mercury[21].
We could also explain the correct gravitational bending of light. Infact in Newtonian theory also we obtain the bending of light, though the amount is half that predicted by General Relativity[22]. In the Newtonian theory we can obtain the bending from the well known orbital equations,

$$\frac{1}{r} = \frac{GM}{L^2}(1 + ecos\Theta) \tag{21}$$

where M is the mass of the central object, L is the angular momentum per unit mass, which in our case is bc, b being the impact parameter or minimum approach distance of light to the object, and e the eccentricity of the trajectory is given by

$$e^2 = 1 + \frac{c^2L^2}{G^2M^2} \tag{22}$$

For the deflection of light α, if we substitute $r = \pm\infty$, and then use (22) we get

$$\alpha = \frac{2GM}{bc^2} \tag{23}$$

This is half the General Relativistic value.
We also note that the effect of time variation on r is given by (cf.ref.[21])

$$r = r_0(1 - \frac{t}{t_0}) \tag{24}$$

Using (24) the well known equation for the trajectory is given by (Cf.[23],[24],[25])

$$u" + u = \frac{GM}{L^2} + u\frac{t}{t_0} + 0\left(\frac{t}{t_0}\right)^2 \tag{25}$$

where $u = \frac{1}{r}$ and primes denote differenciation with respect to Θ.
The first term on the right hand side represents the Newtonian contribution while the remaining terms are the contributions due to (24). The solution of (25) is given by

$$u = \frac{GM}{L^2}\left[1 + ecos\left\{\left(1 - \frac{t}{2t_0}\right)\Theta + \omega\right\}\right] \tag{26}$$

where ω is a constant of integration. Corresponding to $-\infty < r < \infty$ in the Newtonian case we have in the present case, $-t_0 < t < t_0$, where t_0 is large and infinite for practical purposes. Accordingly the analogue of the reception of light for the observer, viz., $r = +\infty$ in the Newtonian case is obtained by taking $t = t_0$ in (26) which gives

$$u = \frac{GM}{L^2} + ecos\left(\frac{\Theta}{2} + \omega\right) \tag{27}$$

Comparison of (27) with the Newtonian solution obtained by neglecting terms $\sim t/t_0$ in equations (24),(25) and (26) shows that the Newtonian Θ is replaced by $\frac{\Theta}{2}$, whence the deflection obtained by equating the left side of (27) to zero, is

$$cos\Theta\left(1 - \frac{t}{2t_0}\right) = -\frac{1}{e} \tag{28}$$

where e is given by (22). The value of the deflection from (28) is twice the Newtonian deflection given by (23). That is the deflection α is now given not by (23) but by the correct formula,

$$\alpha = \frac{4GM}{bc^2},$$

We now come to the problem of galactic rotational curves (cf.ref.[22]). We would expect, on the basis of straightforward dynamics that the rotational velocities at the edges of galaxies would fall off according to

$$v^2 \approx \frac{GM}{r} \tag{29}$$

However it is found that the velocities tend to a constant value,

$$v \sim 300km/sec \tag{30}$$

This has lead to the postulation of dark matter. We observe that from (24) it can be easily deduced that

$$a \equiv (\ddot{r}_o - \ddot{r}) \approx \frac{1}{t_o}(t\ddot{r}_o + 2\dot{r}_o) \approx -2\frac{r_o}{t_o^2} \tag{31}$$

as we are considering infinitesimal intervals t and nearly circular orbits. Equation (31) shows (Cf.ref[21] also) that there is an anomalous inward acceleration, as if there is an extra attractive force, or an additional central mass.

So,

$$\frac{GMm}{r^2} + \frac{2mr}{t_o^2} \approx \frac{mv^2}{r} \tag{32}$$

From (32) it follows that

$$v \approx \left(\frac{2r^2}{t_o^2} + \frac{GM}{r}\right)^{1/2} \tag{33}$$

From (33) it is easily seen that at distances within the edge of a typical galaxy, that is $r < 10^{23}cms$ the equation (29) holds but as we reach the edge and beyond, that is for $r \geq 10^{24}cms$ we have $v \sim 10^7cms$ per second, in agreement with (30).

Thus the time variation of G given in equation (20) explains observation without invoking dark matter.
Interestingly a background Zero Point Field of the type discussed above, is associated with a cosmological constant in General Relativity[26]. We can reconcile this latter view with the above considerations. For this we observe that the variation in G, is small so that over a small period of time the General Relativistic equations hold approximately. Thus we have

$$\ddot{R}(t) = -4\pi\rho(t)GR(t)/3 + \Lambda R(t)/3 \tag{34}$$

In (34) we use equation (20), to get on using the above considerations

$$\Lambda \sim \frac{G\rho}{\sqrt{N}} \tag{35}$$

On the other hand the Zero Point Field leads to a cosmological constant (Cf.ref.[26])

$$\Lambda \sim G < \rho_{vac} > \tag{36}$$

In the above fluctuational cosmological picture, as $\sqrt{N}$ particles are created we get

$$\rho \sim \sqrt{N}\rho_{vac} \tag{37}$$

(35) and (36) can be seen to be identical upon using (37).
This ofcourse should not be surprising, because in both cases we have effectively a cosmological constant which is a manifestation of vaccuum energy.

4 Comments

It must be mentioned that the value of the neutrino mass as deduced in equation (12) rules out the neutrino as a candidate for dark mass, so that there is no contradiction with the observed ever continuing expansion of the universe. It must also be mentioned that the value of the cosmological constant from vacuum energy as deduced by Zeldovich (Cf.ref.[26]) was adhoc and unclear. The effective cosmological constant which we have deduced, however, is consistent.
Interestingly, by reversing the steps in Section 3 we can conclude that a small cosmological constant would imply a variable G.

It may be mentioned that what was called the ether and later the quantum vacuum has been the concept that has survived the whole of the twentieth Century, through the works of Physicists like Dirac[27], Vigier[28], Nelson[29], Prigogine[30], and more recently through the works of Rueda and co-workers[31], the author[5] and even string theoriests like Wilzeck.
We also remark that the considerations of Section 2 (Cf. equations (1) and (2)), show that a Fermion while spread out is localized to within the Compton wavelength. On the other hand the neutrino can be considered to be a truly point particle–the double connectivity of the space, the divide between the region within the Compton wavelength of "negative energy" solutions, and the region without disappears. The neutrino is the divide between Fermions and Bosons.
Finally, it may be mentioned that such a space time cut off is at the heart of a fractal picture of space time, studied by Nottale, Ord, El Naschie, the author and others (Cf.ref.[32] and references therein).

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